RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIRST SEMESTER END SEM EXAMINATION, MARCH 2021
FIRST YEAR [BATCH 2020-23]

Date : 24/03/2021 **MATHEMATICS**
Time : 11.00 am – 1.00 pm **Paper** : MACT 1 Full Marks:50

Instructions to the Candidates

- Write your **College Roll No, Year, Subject** & **Paper Number** on the top of the **Answer Script**.
- Write your **Name, College Roll No, Year, Subject** & **Paper Number** on the **text box of your e-mail**.
- Read the instructions given at the beginning of each paper/group/unit carefully.
- Only handwritten (by blue/black pen) answer-scripts will be permitted.
- Try to answer all the questions of a single group/unit at the same place.
- All the pages of your answer script must be numbered serially by hand.
- In the last page of your answer-script, please mention the total number of pages written so that we can verify it with that of the scanned copy of the script sent by you.
- For an easy scanning of the answer script and also for getting better image, students are advised to write the answers in single side and they must give a minimum 1 inch margin at the left side of each paper.
- After the completion of the exam, scan the entire answer script by using Clear Scan: Indy Mobile App OR any other Scanner device and make a **single PDF file (Named as your College Roll No)** and send it to [redacted]

Group A

(All the notations have their usual meaning)

Answer any **five** questions from question no. $1 - 7$. [5 x 5 = 25 marks]

1. Let f, g, h are three functions such that $f : A \to B$, $g : B \to C$ and $h : C \to A$. Show that if $g \circ f$ and $h \circ g$ are bijective functions then f, g, h are also bijective. [5]

2. On the set $G := (\mathbb{R} \setminus \{0\}) \times \mathbb{R}$; define a law of composition $(*)$ by

$$(a, b) * (c, d) := (ac, bc + d).$$

Show that G is a non-abelian group. [5]

3. Let G be an abelian group and $a, b \in G$ are distinct elements of order 2. Show that $\{e, a, b, ab\}$ forms a subgroup of G that is not cyclic (where e is the identity element of G). [5]

4. Let $H = \{\beta \in S_5 | \ \beta(1) = 1 \ ; \ \beta(3) = 3 \ and \ \beta(4) = 4\}$.
Prove that H is a subgroup of S_5. How many elements are in H? Also find the number of elements in H when S_5 is replaced by S_7. [3 + 1 + 1]

5. (a) Prove that no group can have exactly two elements of order 2. [3]
   (b) If a and b are distinct group elements then show that either $a^2 \neq b^2$ or $a^3 \neq b^3$. [2]

6. (a) Suppose a and b belong to a group G such that a has odd order and $aba^{-1} = b^{-1}$. Show that $b^2 = e$. [3]
   (b) Show that an even order group must have an element of even order. [2]

7. Show that an infinite group must have an infinite number of subgroups. [5]

Group B

Answer any **two** questions from question no. 8 – 10. [12.5 x 2 = 25 marks]

8. (a) If m is a non-square positive integer then show that there does not exist a rational number r such that r^2 =m. [4.5]
   (b) Examine whether $S = \{x \in \mathbb{R} : 4\cos^3 x - 3\cos x \neq 0\}$ is an open set or not in $\mathbb{R}$. [4]
   (c) Find the derived set of $S = \left\{\frac{3}{n} : n \in \mathbb{N}\right\}$. [4]

9. (a) Show that the interval $(0, 2)$ in $\mathbb{R}$ is not denumerable. [4.5]
   (b) Show that the sequence $\left\{\sqrt{3}, \ \sqrt{3+\sqrt{3}}, \ \sqrt{3+\sqrt{3+\sqrt{3}}}, \ldots\right\}$ converges to the positive root of $x^2 - x - 3 = 0$. [4]
   (c) If $f : A \to B$ and $g : B \to C$ be two functions such that $g \circ f$ is surjective , then show that g is surjective. [4]

10. (a) Test the convergence of the the sequence $\{u_n\}_n$ given by $u_n = \sin \frac{n\pi}{2}$. [5]
   (b) Let $\{u_n\}_n$ be a monotone decreasing sequence in $\mathbb{R}$ having a convergent subsequence with subsequential limit u. Then prove that $\{u_n\}_n$ converges to u. [5]
   (c) Find $\lim\limits_{x \to 0} \dfrac{1 - \cosh x}{x^2}$. [2.5]

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